Modeling and Analysis of the Role of Energy Storage for Renewable Integration: Power Balancing

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***Abstract*—The high variability of renewable energy is a major obstacle toward its increased penetration. Energy storage can help reduce the power imbalance due to the mismatch between the available renewable power and the load. How much can storage reduce this power imbalance? How much storage is needed to achieve this reduction? This paper presents a simple analytic model that leads to some answers to these questions. Considering the multitimescale grid operation, we formulate the power imbalance problem for each timescale as an infinite horizon stochastic control problem and show that a greedy policy minimizes the average magnitude of the residual power imbalance. Observing from the wind power data that in shorter timescales the power imbalance can be modeled as an iid zero-mean Laplace distributed process, we obtain closed form expressions for the minimum cost and the stationary distribution of the stored power. We show that most of the reduction in the power imbalance can be achieved with relatively small storage capacity. In longer timescales, the correlation in the power imbalance cannot be ignored. As such, we relax the iid assumption to a weakly dependent stationary process and quantify the limit on the minimum cost for arbitrarily large storage capacity.**

***Index Terms*—Energy storage, renewable integration, stochastic control.**

I. Introduction

THE rapid increase in world demand for electricity [1, Fig. 72] coupled with the need to reduce the high carbon emissions due to electric power generation from fossil fuel [2, Table 3-7] is driving a dramatic increase in renewable energy generation from wind and solar radiation. The power generated from these sources, however, is intermittent and uncertain, which presents significant challenges to power system operation [3]. One such challenge is power imbalance: when renewable generation falls short of meeting the demand, more conventional generation from combined-cycle combustion and gas turbines is needed, which increases power system operation cost and offsets some of the environmental benefits of renewable energy [4]; when renewable generation exceeds demand, the excess power generated must be curtailed.

In addition to using conventional generation, renewable energy variability can be mitigated architecturally via geographical generation diversity [3] and renewable resource diversity [5], and operationally using demand-response [6] and energy storage [7], [8]. In particular, storage can help reduce *both* the amount of conventional generation needed when renewable generation cannot meet the demand and curtailment when there is excess renewable generation. How much can storage help reduce power imbalance, and how much of it is needed to achieve this reduction?

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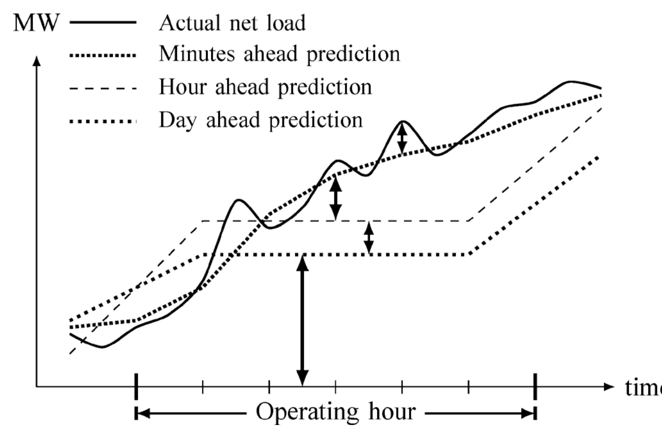

Fig. 1. Illustration of multitimescale electric power system operation.

In this paper, we provide some answers to these questions using a simple analytic approach. Our aim is to establish limits on the benefits of storage rather than analyze the operation of any particular power system with storage. Hence, we do not explicitly consider the fixed or operating costs of storage, or any economic benefit such as arbitrage (e.g., see [9]). Nevertheless, our analysis applies to the case in which the utility company owns and operates the storage.

We assume a power system operating in a multitimescale fashion as follows; see Fig. 1.

- Each day, an hourly prediction of the net load (difference between load and renewable generation) for the next day is made. Base generation and bulk storage are scheduled to meet this prediction.
- Each hour, a refined prediction of the net load in the next hour is made. Peaking generation and storage are scheduled to meet the difference between the day ahead and hour ahead prediction.
- Every few minutes, a prediction of the net load in the next few minutes is made. Fast-ramping generation and storage are scheduled to balance the difference between the minutes ahead and hour ahead prediction.
- The scheduled generation and storage are operated at real time. The deviation of actual net load from the minutes ahead prediction is matched by additional fast-ramping generation and fast-response storage.

To model the multitimescale operation, we formulate the power imbalance problem for each timescale separately as an infinite horizon stochastic control problem. The input for this problem is the *net renewable power imbalance* (or power imbalance in short) process, which is the difference between the predicted net renewable generation (the negative of the net load in Fig. 1) at the next timescale and the predicted net renewable generation at this timescale (possibly with a constant offset). For example, the *hour ahead power imbalance* process is the difference between the minutes ahead prediction and the hour ahead prediction; the *minutes ahead power imbalance* process is the difference between the actual net renewable generation and the minutes ahead prediction, that is, it is the minutes ahead prediction error process. The controls are the storage charging and discharging operations. We refer to the sum of the power imbalance process, the negative of the storage charging power process, and the storage discharging power process as the *residual power imbalance* process. We aim to find the storage control policy that minimizes the long-term average of the magnitude of the residual power imbalance process.

The paper establishes the following results.

- In Section III, we show that a greedy policy minimizes the average magnitude of the residual power imbalance for an arbitrary net renewable power imbalance process (Theorem 1).
- In Section IV, we observe from the NREL and BPA wind power datasets [10], [11] that in the short timescale (minutes), the wind power generation prediction error is close to Laplace distributed. Assuming good prediction and to make the analysis tractable, we model the net renewable power imbalance process in this regime by an independent and identically distributed (iid) zero-mean Laplace distributed process. This allows us to obtain closed form expressions for the minimum cost function (Proposition 1) and the stationary distribution of the stored power sequence (Proposition 2) for unconstrained rated storage input and output powers. We show that most of the reduction in power imbalance is achieved using relatively small storage capacity and corroborate the results numerically. We also show that most of the difference between the analytic and numerical results can be attributed to the iid assumption rather than the Laplace assumption.
- In Section V, we observe that for long timescales (hour ahead) the iid assumption leads to poor results. To attempt to capture the correlation in the power imbalance process, we model it by a weakly dependent stationary process with possibly non-zero mean. We quantify the asymptotic benefits of storage for arbitrarily large storage capacity in terms of the rated input and output powers, the round-trip efficiency, and the first-order distribution of the power imbalance process (Proposition 3). This generalizes and extends the corresponding result under the iid Laplace assumption. We show that this analytic result corroborates extremely well with the numerical results using the NREL dataset.

In [12], we reported preliminary results assuming iid zero-mean Laplace distributed power imbalance and considered only the negative residual power imbalance in the cost function. This paper generalizes and extends the results in [12] in several directions. First, we include the positive residual power imbalance (curtailment) in the cost function. Second, we generalize the asymptotic result to weakly dependent stationary power imbalance processes. We also include detailed proofs that establish the results in [12] as special cases.

The literature on energy storage for renewable integration is quite large (e.g., see [7] and references therein). The most related previous work to this paper are [13]–[15]. In [13], the optimal power flow problem with energy storage is formulated assuming deterministic load with the objective of minimizing the total quadratic generation cost. The resulting optimization problem is not convex in general. However, the dual problem of the convex relaxation is shown to have zero duality gap under certain conditions on the admittance matrix. By comparison, we consider a stochastic setting and formulate an infinite horizon stochastic control problem. Also, our objective is not necessarily optimal power flow, but rather quantifying the limits on the benefit of storage for reducing power imbalance. In [14], the grid operator is assumed to own the energy storage and wishes to minimize the long-term average generation cost, which is an increasing convex function of the load. The load is modeled as an occupation process of an $M/M/\infty$ service queue. A one-threshold policy is shown to be asymptotically optimal for energy storage with perfect round-trip efficiency and infinite capacity. By comparison, we consider imperfect round-trip storage efficiency and show that the average cost is quite sensitive to this efficiency. In [15], an end-user is assumed to own the energy storage and wishes to minimize the discounted cost of purchasing power from the grid. The demand and price processes are assumed to be homogeneous, possibly correlated Markov processes. The optimal control policy is shown to be a stationary two-threshold policy. In this paper, we wish to minimize the average magnitude of the residual power imbalance process, which includes the positive residual power imbalance. This is different from the model in [15] in which excess power can be sold. We note that none of the previous papers obtain limits or closed form expressions for the cost functions or stored power distribution.

II. Problem Formulation

We assume a single-bus electric power system with storage operating in a multitimescale fashion as illustrated in the introduction. To simplify the analysis, we assume that time is slotted into τ-hour intervals, with constant power over each interval, i.e., we ignore generation and load variations within each time interval. In each timescale, we denote the power imbalance at time $i = 1, 2, \ldots$ by $\tau\Delta_i$ MW-h. The power imbalance sequence $\Delta_1, \Delta_2, \ldots$ is in general random with possibly some known statistics. We characterize energy storage by the following parameters.

- The *energy storage capacity* $\tau S_{\max}$ MW-h is the maximum amount of energy that can be stored, where $S_{\max}$ is referred to as the *power storage capacity*.
- The *stored power* at the beginning of time slot $i = 1, 2, \ldots$ is denoted by $S_i \le S_{\max}$.
- The *rated storage input power* $C_{\max}$ MW is the maximum input (charging) power. If the power imbalance $\Delta_i \ge 0$, then the charging power at time $i = 1, 2, \ldots$ denoted by C_i satisfies $0 \le C_i \le \min\{C_{\max}, \Delta_i\}$. If the power imbalance $\Delta_i < 0$, then $C_i = 0$.
- The *rated storage output power* $D_{\max}$ MW is the maximum output (discharging) power. If the power imbalance

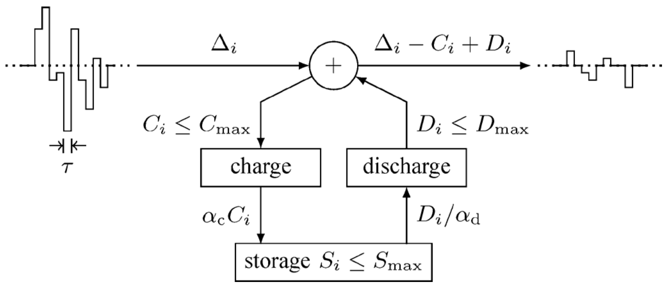

Fig. 2. Illustration of the single-bus power system. The input is the net renewable power imbalance Δ_i, and the output is the residual power imbalance $\Delta_i - C_i + D_i$.

$\Delta_i \leq 0$, then the discharging power at time $i = 1, 2, \ldots$ denoted by D_i satisfies $0 \leq D_i \leq \min\{D_{\max}, -\Delta_i\}$. If the power imbalance $\Delta_i > 0$, then $D_i = 0$.

- The *charging efficiency* α_c is the ratio of the charged power to the input power. The *discharging efficiency* α_d is the ratio of the output power to the discharged power. The *round-trip efficiency* is $\alpha = \alpha_c \alpha_d$.
- We assume that the *storage efficiency*, which is the fraction of stored power retained over a time slot, is equal to one.

Using the above definitions, we can express the dynamics of the stored power as

$$S_{i+1} = S_i + \alpha_c C_i - \frac{1}{\alpha_d} D_i, \qquad \text{for } i = 1, 2, \ldots$$

with the constraints $0 \leq C_i \leq \min\{C_{\max}, \Delta_i^+, (S_{\max} - S_i)/\alpha_c\}$, and $0 \leq D_i \leq \min\{D_{\max}, \Delta_i^-, \alpha_d S_i\}$, where $x^+ = \max\{x, 0\}$, $x^- = -\min\{x, 0\}$, and S_1 is given.

The *residual power imbalance* at each time i is the power imbalance after the charging/discharging operation of energy storage and is equal to $\Delta_i - C_i + D_i$. Fig. 2 illustrates the power flow in the single-bus system for a given timescale.

We wish to minimize the expected average magnitude of the residual power imbalance by controlling the charging power C_i and discharging power D_i of the energy storage in each time slot $i = 1, 2, \ldots$, where the pair (C_i, D_i) is a function of the *history* $H_i = (S_1, \Delta_1, S_2, \Delta_2, \ldots, S_{i-1}, \Delta_{i-1}, S_i)$. A control policy π is a sequence of these pairs, i.e., $\pi = \{(C_i, D_i) : i = 1, 2, \ldots\}$. A policy is said to be stationary if $\pi_i = \pi_j$ for all i and j.

We are now ready to formulate the infinite horizon stochastic control problem investigated in this paper given as follows:

$$\begin{aligned} \text{minimize} \quad & \mathcal{J}(\pi, S_1) = \limsup_{n \to \infty} \mathrm{E}\left[\frac{1}{n}\sum_{i=1}^{n} |\Delta_i - C_i + D_i|\right] \\ \text{subject to} \quad & S_{i+1} = S_i + \alpha_c C_i - \frac{1}{\alpha_d} D_i \\ & 0 \leq C_i \leq \min\left\{C_{\max}, \Delta_i^+, \frac{S_{\max} - S_i}{\alpha_c}\right\} \\ & 0 \leq D_i \leq \min\left\{D_{\max}, \Delta_i^-, \alpha_d S_i\right\} \end{aligned}$$

where the expectation is over the power imbalance Δ_i, $i = 1, 2, \ldots$. We denote the optimal policies by π^*.

III. Optimal Control Policy

We show that, under the setup in the previous section, a simple stationary greedy policy is optimal.

TABLE I
Optimal Policy in Theorem 1

π^*	C_i^*	D_i^*
$\frac{S_{\max} - S_i}{\alpha_c} \leq \min\{\Delta_i, C_{\max}\}$	$\frac{S_{\max} - S_i}{\alpha_c}$	0
$C_{\max} \leq \min\left\{\Delta_i, \frac{S_{\max} - S_i}{\alpha_c}\right\}$	$C_{\max}$	0
$0 \leq \Delta_i < \min\left\{\frac{S_{\max} - S_i}{\alpha_c}, C_{\max}\right\}$	Δ_i	0
$\max\{-\alpha_d S_i, -D_{\max}\} \leq \Delta_i < 0$	0	$-\Delta_i$
$\max\{\Delta_i, -D_{\max}\} < -\alpha_d S_i$	0	$\alpha_d S_i$
$\max\{\Delta_i, -\alpha_d S_i\} < -D_{\max}$	0	$D_{\max}$

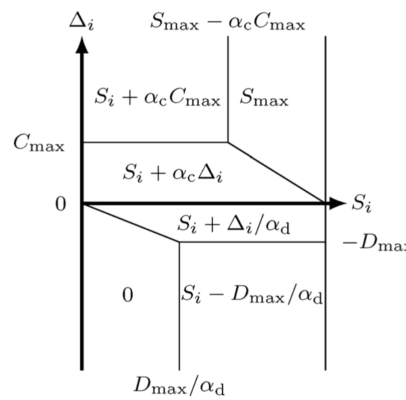

Fig. 3. Illustration of the optimal policy π^* in Theorem 1. The value in each region corresponds to the stored power at the end of slot i when the stored power and prediction error at the beginning of this slot are S_i and Δ_i, respectively.

Theorem 1: The optimal policy π^* is given in Table I and illustrated in Fig. 3. It is stationary.

When the power imbalance is positive, the optimal policy charges the energy storage by as much excess power as possible. When the power imbalance is negative, there is excess load, and the optimal policy discharges the energy storage to satisfy as much excess load as possible.

To prove Theorem 1, consider the finite horizon counterpart. Define the cost-to-go function

$$J_k(\pi, H_k) = \mathrm{E}\left[\sum_{i=k}^{n} |\Delta_i - C_i + D_i| \,\middle|\, H_k\right].$$

Let π^* be the policy achieving $\inf_\pi J_k(\pi, H_k)$. For a fixed pair (H_{k-1}, Δ_{k-1}), we will use the shorter notation $J_k(\pi^*, S_k)$ in place of $J_k(\pi^*, H_{k-1}, \Delta_{k-1}, S_k)$. In the following lemma, we establish key properties of the cost-to-go function. The remainder of the proof of Theorem 1 is given in Appendix A.

Lemma 1: The minimum cost-to-go function must satisfy the following conditions.

1) $J_k(\pi^*, S_k + c) \leq J_k(\pi^*, S_k) + c/\alpha_c$ for $0 \leq c \leq S_{\max} - S_k$. If power imbalance $c/\alpha_c > 0$, then the cost of not charging is c/α_c, but the reduction in future cost is at most c/α_c.
2) $J_k(\pi^*, S_k + c) \geq J_k(\pi^*, S_k) - \alpha_d c$ for $0 \leq c \leq S_{\max} - S_k$. If power imbalance $-\alpha_d c < 0$, then the cost of not discharging is $\alpha_d c$, but the reduction in future cost is at most $\alpha_d c$.

Proof of Lemma 1: To prove part 1, given the optimal policy π^*, we find another policy π such that $J_k(\pi, S_k + c) \leq J_k(\pi^*, S_k) + c/\alpha_c$. Let H_k and H_k^* be the history sequences under the policy π and π^*, respectively. For fixed (H_{k-1}, Δ_{k-1}), consider $H_k = (H_{k-1}, \Delta_{k-1}, S_k + c)$ and $H_k^* = (H_{k-1}, \Delta_{k-1}, S_k)$. Let

$$D_i = D_i^*, \quad C_i = \min\left\{C_i^*, \frac{S_{\max} - S_i}{\alpha_c}\right\} \leq C_i^*. \quad (1)$$

The policy π is illustrated in Fig. 4(a). By induction, the stored power under policy π is always higher than that under policy π^*. Consider the cost difference

$$\begin{aligned} &|\Delta_i - C_i + D_i| - |\Delta_i - C_i^* + D_i^*| \\ &\quad = |\Delta_i - C_i + D_i^*| - |\Delta_i - C_i^* + D_i^*| \\ &\quad \leq C_i^* - C_i = \frac{1}{\alpha_c}((S_i - S_i^*) - (S_{i+1} - S_{i+1}^*)). \end{aligned}$$

The difference between the cost-to-go functions of policies π and π^* can be upper bounded as

$$\begin{aligned} &J_k(\pi, S_k + c) - J_k(\pi^*, S_k) \\ &\quad \leq \mathrm{E}\left[\sum_{i=k}^{n} \frac{1}{\alpha_c}((S_i - S_i^*) - (S_{i+1} - S_{i+1}^*)) \middle| H_k\right] \\ &\quad = \frac{1}{\alpha_c}\mathrm{E}\left[(S_k - S_k^*) - (S_{n+1} - S_{n+1}^*) \middle| H_k\right] \leq \frac{c}{\alpha_c}. \end{aligned}$$

Thus, $J_k(\pi^*, S_k + c) \leq J_k(\pi, S_k + c) \leq J_k(\pi^*, S_k) + c/\alpha_c$.

To prove part 2 of the lemma, given the optimal policy π^*, we find a policy π such that $J_k(\pi, S_k) \leq J_k(\pi^*, S_k + c) + \alpha_d c$. For fixed (H_{k-1}, Δ_{k-1}), consider $H_k = (H_{k-1}, \Delta_{k-1}, S_k)$ and $H_k^* = (H_{k-1}, \Delta_{k-1}, S_k + c)$. Let

$$C_i = C_i^* \qquad D_i = \min\{D_i^*, \alpha_d S_i\} \leq D_i^*. \quad (2)$$

The policy π is illustrated in Fig. 4(b). By induction, the stored power under policy π is always lower than the stored power under policy π^*. Consider the cost difference

$$\begin{aligned} &|\Delta_i - C_i + D_i| - |\Delta_i - C_i^* + D_i^*| \\ &\quad = |\Delta_i - C_i^* + D_i| - |\Delta_i - C_i^* + D_i^*| \\ &\quad \leq D_i^* - D_i = \alpha_d((S_i^* - S_i) - (S_{i+1}^* - S_{i+1})). \end{aligned}$$

The difference between the cost-to-go functions of policies π and π^* can be upper bounded as

$$\begin{aligned} &J_k(\pi, S_k) - J_k(\pi^*, S_k + c) \\ &\quad \leq \mathrm{E}\left[\sum_{i=k}^{n} \alpha_d\left((S_i^* - S_i) - (S_{i+1}^* - S_{i+1})\right) \middle| H_k\right] \\ &\quad = \alpha_d \mathrm{E}\left[(S_k^* - S_k) - (S_{n+1}^* - S_{n+1}) \middle| H_k\right] \leq \alpha_d c. \end{aligned}$$

Thus, $J_k(\pi^*, S_k) \leq J_k(\pi, S_k) \leq J_k(\pi^*, S_k + c) + \alpha_d c$. ■

To illustrate the optimal policy π^*, we use the simulated Western Wind Dataset from NREL [10] for the 50 highest power density offshore sites in California. The wind power data are sampled every 10 min. We choose the mean of the aggregate wind power from these 50 sites as the base power, i.e., it is one per unit. We assume that the load is accurately predicted, and thus the wind power is the only source of power imbalance. Since the dataset does not include forecast data, we use a simple linear predictor. Fig. 5 illustrates the hour ahead power imbalance and the residual power imbalance for two days.

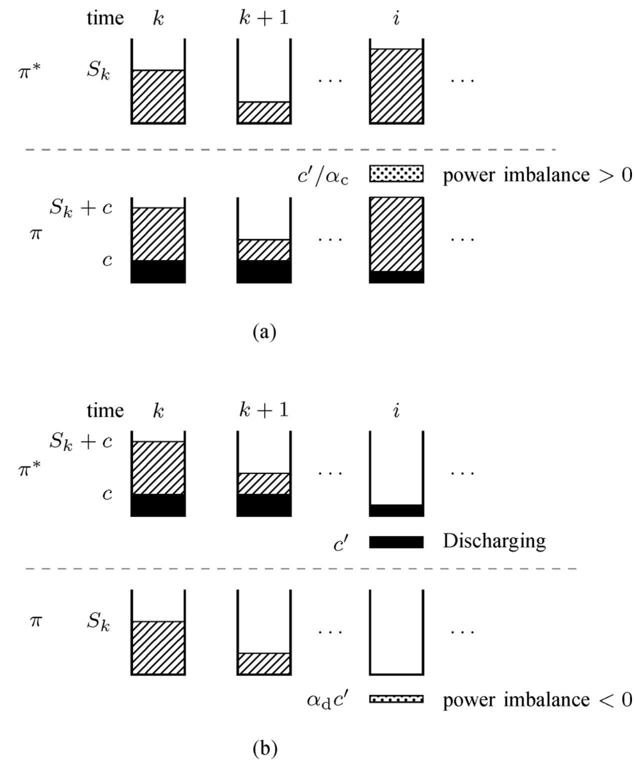

Fig. 4. Illustration for the proof of Lemma 1. (a) The policy π follows the optimal policy π^* if the stored power is below capacity and has additional cost (positive residual power imbalance) when the storage is full. (b) The policy π follows the optimal policy π^* if the storage is not empty and has additional cost (negative residual power imbalance) when the storage is empty.

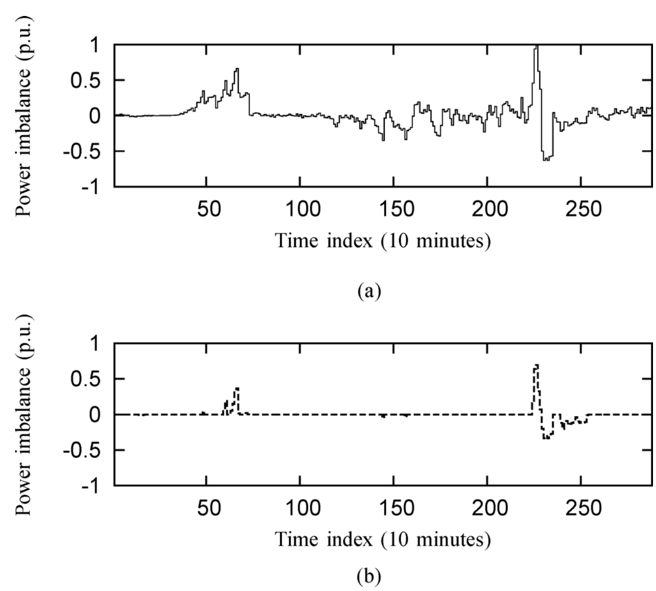

Fig. 5. Illustration of the hour ahead power imbalance and the residual power imbalance for two days assuming unlimited $S_{\max}$, $C_{\max} = D_{\max} = 0.3$, and $\alpha = \alpha_c^2 = \alpha_d^2 = 70\%$, The hour ahead prediction uses the persistent predictor, that is, the prediction is the average wind power of the past hour. The 10-min ahead prediction uses the linear predictor based on the 6 samples from the past hour and optimized for the one-year data in 2004. The average magnitude of power imbalance is 0.094 per unit, and the average magnitude of residual power imbalance is 0.022 per unit. (a) Power imbalance (NREL). (b) Residual power imbalance (NREL).

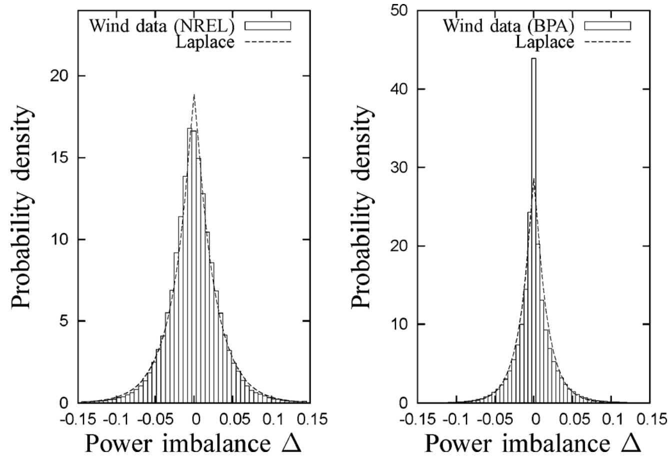

Fig. 6. Empirical distributions of the 10- and 5-min-ahead power imbalance for the NREL and BPA wind power datasets, respectively, and the best fit Laplace distributions. The standard deviations for the NREL and BPA datasets are 0.037 and 0.027 per unit, respectively.

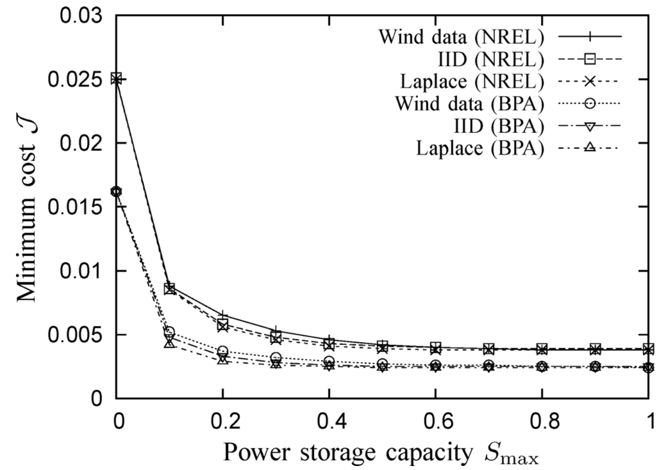

Fig. 7. Minimum cost function versus power storage capacity at the minutes ahead timescale for round-trip efficiency $\alpha = \alpha_c^2 = \alpha_d^2 = 70\%$.

IV. IID Laplace Power Imbalance Process

In the previous section, we formulated the power imbalance problem as an infinite horizon stochastic control problem and found the optimal policy minimizing the average residual power imbalance for arbitrary power imbalance process at each timescale. In exploring the NREL and BPA wind datasets [11], [16], we find that the first-order distributions of their 10- and 5-min ahead prediction errors, respectively, can be reasonably approximated by Laplace(λ) probability density function (pdf) $f_\Delta(\delta) = \lambda e^{-\lambda|\delta|}/2$ (see Fig. 6). Note, however, that the Laplace assumption fits the NREL dataset better than the BPA dataset for which the empirical pdf is more peaky than the Laplace distribution.

To simplify the analysis and as a possibly good model for error when the prediction is performed well, we further assume that the prediction error sequence is iid. This allows us to obtain a closed form expression for the minimum cost function when the rated input and output powers are unconstrained, i.e., $\alpha_c C_{\max} = D_{\max}/\alpha_d = S_{\max}$.

Proposition 1: The minimum average magnitude of the residual power imbalance under the iid Laplace assumption is

$$\mathcal{J}(\pi^*, S_1) = \frac{1}{2\lambda}\left(\frac{(1-\alpha)(1+e^{-\xi S_{\max}})}{1-\alpha e^{-\xi S_{\max}}}\right)$$

where $\xi = (1/\alpha_c - \alpha_d)\lambda/2$.

We can also find a closed form expression for the stationary distribution of the stored power sequence.

Proposition 2: The cumulative distribution function (cdf) of the stationary distribution of the stored power sequence under the optimal policy in Theorem 1 and the iid Laplace assumption is

$$F_S(s) = \frac{1-0.5(1+\alpha)e^{-\xi s}}{1-\alpha e^{-\xi S_{\max}}}$$

for $0 \le s < S_{\max}$, $F_S(s) = 0$ for $s < 0$, and $F_S(s) = 1$ for $s \ge S_{\max}$.

The proofs for these two propositions are given in Appendix B.

Note that with no storage, the minimum cost is $1/\lambda$, and with unlimited power storage capacity, the minimum cost is $(1-\alpha)/2l$. By Proposition 1, we have

$$S_{\max} = \frac{1}{\xi}\ln\left(\frac{2\alpha\lambda\mathcal{J}(\pi^*, S_1) + (1-\alpha)}{2l\mathcal{J}(\pi^*, S_1) - (1-\alpha)}\right).$$

For a typical round-trip efficiency $50\% \le \alpha \le 95\%$, 80% of the possible reduction in cost, i.e.,

$$\mathcal{J}(\pi^*, S_1) = \frac{1}{\lambda} - 0.8\left(\frac{1}{\lambda} - \frac{1-\alpha}{2l}\right) = \frac{0.6-0.4\alpha}{\lambda}$$

is achieved with power storage capacity less than five standard deviations of the power imbalance process, where the standard deviation of a Laplace(λ) random variable is $\sqrt{2}/\lambda$.

Fig. 7 plots the minimum costs using the analytic result of Proposition 1 and the NREL and BPA wind power datasets. With storage capacity $S_{\max} = 1$ per unit, the energy storage reduces the average magnitude of the power imbalance by 84.8% and 85.2% for the NREL and BPA datasets, respectively. Eighty percent of this reduction can be achieved by a storage capacity of $S_{\max} =$ 0.13 and 0.10 per unit, respectively, which are only four empirical standard deviations of the NREL and BPA datasets. Note that the minimum cost using the analytic result differs by at most 13.8% and 21.6% from those using the NREL and BPA datasets, respectively. To determine the contribution in this difference due to the iid assumption, we generate iid power imbalance sequences according to the empirical first-order distributions of the NREL and BPA datasets. In Fig. 7, we compare the minimum costs using these iid sequences to those using the datasets. The minimum costs for the iid sequences differ by at most 10.8% and 12.5% from those using the NREL and BPA datasets, respectively, hence the iid assumption is a larger contributor to the difference in the cost between the analytic and numerical results than is the Laplace assumption. Note that the Laplace assumption contributes more to the difference for the BPA dataset case than to that for the NREL dataset case because its empirical pdf is not as close to Laplace as that for the NREL dataset.

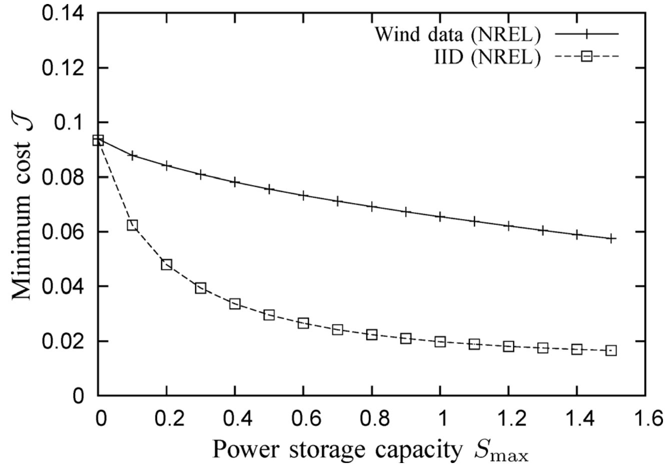

Fig. 8. Minimum cost function versus power storage capacity at the hour ahead timescale for round-trip efficiency $\alpha = \alpha_c^2 = \alpha_d^2 = 70\%$. The standard deviation of the hour ahead power imbalance is 0.149 per unit.

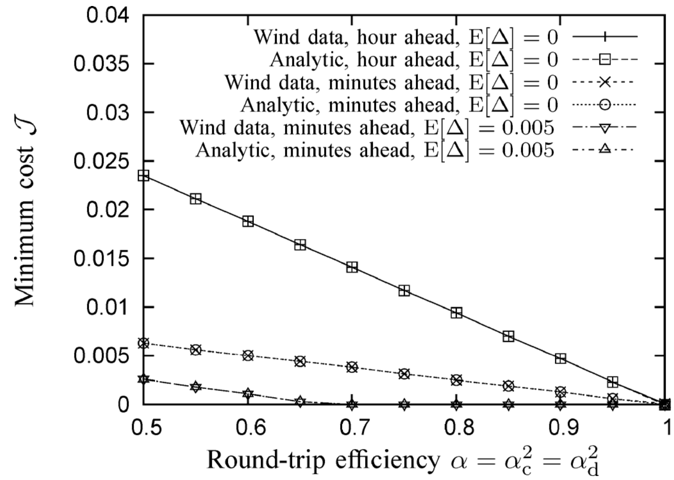

Fig. 9. Minimum cost function versus the round-trip efficiency for unlimited storage capacity and rated input and output powers at the minutes ahead and hour ahead timescales, where "Wind data" represents the NREL dataset, and "Analytic" represents the result of Proposition 3 with empirical estimates of $\mathrm{E}[\Delta^+]$ and $\mathrm{E}[\Delta^-]$ from the dataset.

V. Stationary Power Imbalance Process

In the previous section, we observed that the power imbalance process at short timescales can be reasonably modeled as an iid process. However, for longer timescales, the example in Fig. 8 demonstrates that the temporal correlation of the power imbalance cannot be ignored. To attempt to capture this correlation, in this section we relax the iid assumption and model the power imbalance process by a κ-*weak dependent* stationary process [17] with $\kappa_k = \mathcal{O}(1/k)$, i.e., $|\mathrm{Cov}\,(g(\Delta_i), h(\Delta_j))| \leq \mathrm{Lip}(g)\mathrm{Lip}(h)\kappa_k$ for all functions g, h, and indices $i + k \leq j$, where $\mathrm{Lip}\,(g) = \sup_{x \neq y} |g(x) - g(y)|/\|x - y\|_1$ and $\mathrm{Lip}(h)$ is defined similarly. Note that an iid process is a special case of the κ-weak dependent stationary process with $\kappa_k = \mathcal{O}(1/k)$. We further assume an arbitrary first-order pdf $f_\Delta(\delta)$ with finite mean $\mathrm{E}\,(\Delta) = \mu$ and variance $\mathrm{Var}\,(\Delta) = \sigma^2$.

Under this relaxed assumption on the power imbalance process, we are able to quantify the minimum cost function for extreme cases. With no storage, i.e., $S_{\max} = 0$, the minimum cost $\mathcal{J}(\pi^*, S_1) = \mathrm{E}\,[|\Delta|]$. This is also an upper bound on the minimum cost for general $S_{\max}$. At the other extreme, when the power storage capacity $S_{\max}$ is unlimited, the minimum cost is as follows.

Proposition 3: For unlimited $S_{\max}$, if the power imbalance process is a κ-weak dependent stationary process with $\kappa_k = \mathcal{O}(1/k)$, then the minimum average magnitude of the residual power imbalance is

$$\begin{aligned}\mathcal{J}(\pi^*, S_1) &= \mathrm{E}\,[|\Delta|] - \mathrm{E}\,[\min\{\Delta^+, C_{\max}\}] \\ &\quad - \min\{\alpha\,\mathrm{E}\,[\min\{\Delta^+, C_{\max}\}],\ \mathrm{E}\,[\min\{\Delta^-, D_{\max}\}]\}.\end{aligned}$$

The proof of this proposition is given in Appendix C.

Note that the minimum cost depends only on the first-order pdf of the power imbalance process, the round-trip efficiency, and the rated input and output powers. Under the iid Laplace assumption, this result reduces to the result in Proposition 1 for unlimited $S_{\max}$ and unconstrained $C_{\max}$ and $D_{\max}$. Other statistical models of the wind power forecast error, such as in [18]–[20], can be similarly used to obtain more explicit expressions of the minimum cost.

To compare the minimum costs with no storage and with unlimited storage, we consider the following four regions of the power imbalance space: $\Delta < -D_{\max}$, $-D_{\max} \leq \Delta < 0$, $0 < \Delta \leq C_{\max}$, and $\Delta > C_{\max}$. For $S_{\max} = 0$, the cost is the sum of the power imbalance in all these regions. For unlimited $S_{\max}$, the power imbalance costs in the regions $\Delta < -D_{\max}$ and $\Delta > C_{\max}$ are the same as the corresponding costs for $S_{\max} = 0$ because of the rated output and input power constraints. However, the power imbalance for $0 < \Delta \leq C_{\max}$ is used to charge the storage, hence does not contribute to the cost. For the fourth region $-D_{\max} \leq \Delta < 0$, by Proposition 3, all of the stored power except the loss due to the imperfect round-trip efficiency can be used to satisfy as much of the power imbalance in this region as possible over the long term. The remaining power imbalance in this region, if any, also contributes to the cost.

Fig. 9 illustrates the minimum cost function for the NREL dataset with $50\% \leq \alpha \leq 100\%$ and unlimited storage capacity and rated input and output powers. Note that the numerical results corroborate well with the analytic result of Proposition 3. For perfect round-trip efficiency, the energy storage can attain zero residual power imbalance. The results for the BPA dataset are quite similar, hence we do not include them.

VI. Conclusion

We aimed to answer questions concerning the benefits of energy storage for reducing the power imbalance due to renewable generation: how much can energy storage reduce power imbalance, and how much of it is needed to achieve this reduction? To answer these questions, we considered the multi-timescale operation of the grid and formulated the power imbalance problem for each timescale as an infinite horizon stochastic control problem. We obtained analytic results by modeling the power imbalance process at short timescales by an iid Laplace distributed process and at longer timescales by a weakly dependent stationary process. The results lead to the following potentially useful conclusions.

- The iid Laplace distribution, which makes the analysis far more tractable, appears to be a reasonable approximation of the short timescale (5–10 min) prediction error of wind power generation (Propositions 1 and 2).
- Eighty percent of the reduction in the cost function can be achieved using storage capacity less than five standard deviations of the power imbalance (Proposition 1).
- Energy storage can reduce the average positive power imbalance to zero (Proposition 3).
- When the mean of the power imbalance is large, energy storage can reduce the average negative power imbalance to zero (Proposition 3).
- When the mean of the power imbalance is small, energy storage can reduce the average negative power imbalance by the round-trip efficiency times the average positive power imbalance (Proposition 3).

We corroborated our assumptions and analytic results using the NREL and BPA wind power datasets.

Appendix A
Proof for Optimal Control Policy

Proof of Theorem 1: We need to show that, for any policy π, then

$$\begin{aligned} J_k &= |\Delta_k - C_k + D_k| + J_{k+1}(\pi^*, S_{k+1}) \\ &\geq |\Delta_k - C_k^* + D_k^*| + J_{k+1}(\pi^*, S_{k+1}^*) = J_k^* \end{aligned}$$

where $S_{k+1} = S_k + \alpha_{\rm c} C_k - D_k/\alpha_{\rm d}$ and $S_{k+1}^* = S_k + \alpha_{\rm c} C_k^* - D_k^*/\alpha_{\rm d}$. If $\Delta_k \geq 0$, then by definition

$$C_k^* = \min\left\{C_{\max}, \Delta_k^+, \frac{S_{\max} - S_k}{\alpha_{\rm c}}\right\} \geq C_k,$$

and thus $S_{k+1}^* = S_k + \alpha_{\rm c} C_k^* \geq S_k + \alpha_{\rm c} C_k = S_{k+1}$. By Part 1 of Lemma 1,

$$J_k - J_k^* \geq (\Delta_k - C_k) - (\Delta_k - C_k^*) - \frac{S_{k+1}^* - S_{k+1}}{\alpha_{\rm c}} = 0.$$

If $\Delta_k \leq 0$, then by definition

$$D_k^* = \min\{D_{\max}, \Delta_k^-, \alpha_{\rm d} S_k\} \geq D_k,$$

and thus $S_{k+1}^* = S_k - D_k^*/\alpha_{\rm d} \leq S_k - D_k/\alpha_{\rm d} = S_{k+1}$. By Part 2 of Lemma 1,

$$J_k - J_k^* \geq (-\Delta_k - D_k) - (-\Delta_k - D_k^*) - \alpha_{\rm d}(S_{k+1} - S_{k+1}^*) = 0.$$

By induction, we show that the stationary policy π^* is optimal for the finite horizon stochastic control problem. Then, for any $n \in \mathbb{N}$ and policy π, we have $J_1(\pi^*, S_1)/n \leq J_1(\pi, S_1)/n$, and thus

$$\begin{aligned} J(\pi^*, S_1) &= \limsup_{n\to\infty} \frac{1}{n} J_1(\pi^*, S_1) \\ &\leq \limsup_{n\to\infty} \frac{1}{n} J_1(\pi, S_1) = J(\pi, S_1). \end{aligned}$$

■

Appendix B
Proof for Results Under the Laplace Assumption

Proof of Proposition 1: To find the minimum cost $\mathcal{J}(\pi^*, S_1)$ under the Laplace assumption, we need to show that there exist a constant η and a function $w(s)$ satisfying the average cost optimality equation

$$\begin{aligned} \eta + w(s) &= \mathrm{E}\left[|\Delta - C^* + D^*| + w\left(s + \alpha_{\rm c} C^* - \frac{1}{\alpha_{\rm d}} D^*\right)\right] \\ &= \mathrm{E}\left[\Delta - \frac{S_{\max} - s}{\alpha_{\rm c}} + w(S_{\max}); \frac{S_{\max} - s}{\alpha_{\rm c}} \leq \Delta\right] \\ &\quad + \mathrm{E}\left[w(s + \alpha_{\rm c}\Delta); 0 \leq \Delta < \frac{S_{\max} - s}{\alpha_{\rm c}}\right] \\ &\quad + \mathrm{E}\left[w\left(s + \frac{1}{\alpha_{\rm d}}\Delta\right); -\alpha_{\rm d} s \leq \Delta < 0\right] \\ &\quad + \mathrm{E}\left[-\Delta - \alpha_{\rm d} s + w(0); \Delta < -\alpha_{\rm d} s\right] \end{aligned} \tag{3}$$

and $\lim_{n\to\infty} \mathrm{E}\left[w(S_n)/n\right] = 0$ [21], where, for a random variable X and a set A, we define $\mathrm{E}[X; A] = \mathrm{E}[X|A]\,\mathrm{P}(A)$. Then, the average cost is equal to η. Now we verify that, for the policy π^* in Theorem 1, then

$$\begin{aligned} \eta &= \frac{1}{2\lambda}\left(\frac{(1-\alpha)(1+e^{-\xi S_{\max}})}{1-\alpha e^{-\xi S_{\max}}}\right) \\ w(s) &= \frac{(1+\alpha)^2}{\lambda(1-\alpha)}\left(\frac{e^{-\xi(S_{\max}-s)}}{1-\alpha e^{-\xi S_{\max}}}\right) - \left(\frac{\alpha_{\rm d}(1+e^{-\xi S_{\max}})}{1-\alpha e^{-\xi S_{\max}}}\right) s \end{aligned}$$

satisfies (3). Furthermore, $w(s)$ is bounded for $s \in [0, S_{\max}]$, and thus $\lim_{n\to\infty} \mathrm{E}\left[w(S_n)/n\right] = 0$. ■

Proof of Proposition 2: Since the optimal policy π^* in Theorem 1 is stationary, the corresponding stored power sequence is a Markov process

$$S_{i+1} = \begin{cases} S_{\max}, & \text{if } (S_{\max} - S_i)/\alpha_{\rm c} \leq \Delta_i \\ S_i + \alpha_{\rm c}\Delta_i, & \text{if } 0 \leq \Delta_i < (S_{\max} - S_i)/\alpha_{\rm c} \\ S_i + \Delta_i/\alpha_{\rm d}, & \text{if } -\alpha_{\rm d} S_i \leq \Delta_i < 0 \\ 0, & \text{if } \Delta_i < -\alpha_{\rm d} S_i. \end{cases}$$

Let F_i be the cdf of S_i. Then for $0 \leq s < S_{\max}$,

$$\begin{aligned} F_{i+1}(s) &= \int_{-\infty}^{-\alpha_{\rm d}(S_{\max}-s)} \frac{\lambda}{2} e^{\lambda\delta} F_i(S_{\max}) d\delta \\ &\quad + \int_{-\alpha_{\rm d}(S_{\max}-s)}^{0} \frac{\lambda}{2} e^{\lambda\delta} F_i\left(s - \frac{\delta}{\alpha_{\rm d}}\right) d\delta \\ &\quad + \int_0^{s/\alpha_{\rm c}} \frac{\lambda}{2} e^{-\lambda\delta} F_i(s - \alpha_{\rm c}\delta) d\delta \end{aligned} \tag{4}$$

where the limits of the integrals are determined from Fig. 3. For $s < 0$, $F_{i+1}(s) = 0$, and for $s \geq S_{\max}$, $F_{i+1}(s) = 1$. Let

$$F_S(s) = \begin{cases} 0, & \text{if } s < 0 \\ \dfrac{1 - 0.5(1+\alpha)e^{-\xi s}}{1 - \alpha e^{-\xi S_{\max}}}, & \text{if } 0 \leq s < S_{\max} \\ 1, & \text{if } s \geq S_{\max}. \end{cases}$$

It can be verified that (4) is satisfied with $F_i(s) = F_{i+1}(s) = F_S(s)$ for all s. Now we only need to show that the Markov process is irreducible with respect to $F_S(s)$, which implies that the stationary distribution is unique [22, Theorem 4.1]. Let $S_i =$

s, and let $\mathcal{B} = (b_1, b_2) \subset [s, S_{\max}]$ such that $\mathrm{P}\{S \in \mathcal{B}\} > 0$ under the stationary distribution, that is, $b_2 > b_1$. Then

$$\begin{aligned}\mathrm{P}\{S_{i+1} \in \mathcal{B}\} &= \int_{(b_1-s)/\alpha_c}^{(b_2-s)/\alpha_c} \frac{\lambda}{2} e^{-\lambda\delta} d\delta \\ &= \frac{1}{2}\left(e^{-\lambda(b_1-s)/\alpha_c} - e^{-\lambda(b_2-s)/\alpha_c}\right) > 0.\end{aligned}$$

If $\mathcal{B} = \{0\}$, then $P\{S_{i+1} \in \mathcal{B}\} = \int_{-\infty}^{-\alpha_d s} (\lambda/2) e^{\lambda\delta} d\delta = (1/2)e^{-\alpha_d \lambda s} > 0$. Similarly, $\mathrm{P}\{S_{i+1} \in \mathcal{B}\} > 0$ for $\mathcal{B} = (b_1, b_2) \subset [0, s]$ and $\{S_{\max}\}$. Thus, we can generalize $\mathcal{B}$ to any set such that $\mathrm{P}\{S \in \mathcal{B}\} > 0$. Therefore, the Markov process is irreducible. ■

APPENDIX C
PROOF OF PROPOSITION 3

We first establish bounds on the expected stored power.

Lemma 2: Suppose that $S_{i+1} = \max\{S_i + \Theta_i, 0\}$ for $i = 1, 2, \ldots, n$, where $S_1 \geq 0$ is given and Θ_i, $i = 1, 2, \ldots, n$, is a stationary process with mean μ_Θ, variance σ_Θ^2, and autocovariance $\mathrm{Cov}(\Theta_i, \Theta_{i+k}) = \mathcal{O}(1/k)$.

1) If $\mu_\Theta \leq 0$, then $\mathrm{E}[S_{k+1}] = \mathcal{O}(\sqrt{k}(\log k)^{3/2})$.
2) If $\mu_\Theta > 0$, then $\mathrm{E}[S_{k+1}] - k\mu_\Theta = \mathcal{O}(\sqrt{k}(\log k)^{3/2})$.

Proof of Lemma 2: If $\mu_\Theta \leq 0$, then

$$\begin{aligned}&\mathrm{E}[S_{k+1} - S_1] \\ &\quad= \mathrm{E}\left[\max\left\{0,\ S_1 + \sum_{i=1}^k \Theta_i,\ \max_{2\leq j\leq k} \sum_{i=j}^k \Theta_i\right\} - S_1\right] \\ &\quad\leq \mathrm{E}\left[\max_{1\leq j\leq k}\left|\sum_{i=j}^k \tilde{\Theta}_i\right|\right]\end{aligned}$$

where $\tilde{\Theta}_i = \Theta_i - \mu_\Theta$ for $i = 1, 2, \ldots, k$. Note that $\mathrm{E}[\tilde{\Theta}_i] = 0$ and $\mathrm{E}[\tilde{\Theta}_i \tilde{\Theta}_{i+k}] = \mathrm{Cov}(\Theta_i, \Theta_{i+k}) = \mathcal{O}(1/k)$. If $\mu_\Theta > 0$, then

$$\begin{aligned}&\mathrm{E}[S_{k+1} - S_1] - k\mu_\Theta \\ &\quad= \mathrm{E}\left[\max\left\{0,\ S_1 + \sum_{i=1}^k \Theta_i,\ \max_{2\leq j\leq k} \sum_{i=j}^k \Theta_i\right\} - S_1\right] \\ &\qquad - k\mu_\Theta \\ &\quad\leq \mathrm{E}\left[\max_{1\leq j\leq k}\left|\sum_{i=j}^k \tilde{\Theta}_i\right|\right]\end{aligned}$$

and

$$\begin{aligned}\mathrm{E}[S_{k+1} - S_1] - k\mu_\Theta &\geq \mathrm{E}[S_k + \Theta_k - S_1] - k\mu_\Theta \\ &= \mathrm{E}[S_k - S_1] - (k-1)\mu_\Theta \geq 0,\end{aligned}$$

where the last inequality follows by induction. To complete the proof, we only need to show that

$$\begin{aligned}\left(\mathrm{E}\left[\max_{1\leq j\leq k}\left|\sum_{i=j}^k \tilde{\Theta}_i\right|\right]\right)^2 &\leq \mathrm{E}\left[\left(\max_{1\leq j\leq k}\left|\sum_{i=j}^k \tilde{\Theta}_i\right|\right)^2\right] \\ &= \mathcal{O}\left(k(\log k)^3\right),\end{aligned}$$

where the first inequality follows by Jensen's inequality. By the inequality in [23, Corollary A2], we have

$$\begin{aligned}&\mathrm{E}\left[\left(\max_{1\leq j\leq k}\left|\sum_{i=j}^k \tilde{\Theta}_i\right|\right)^2\right] \\ &\quad\leq (\log_2 2k)^2 k \left(\sigma_\Theta^2 + 2\sum_{j=1}^{k-1} |\mathrm{E}[\tilde{\Theta}_1 \tilde{\Theta}_{1+j}]|\right) \\ &\quad= \mathcal{O}(k(\log k)^3).\end{aligned}$$

■

Proof of Proposition 3: Let

$$\Theta_i = \alpha_c \min\{\Delta_i^+, C_{\max}\} - \frac{1}{\alpha_d} \min\{\Delta_i^-, D_{\max}\}.$$

We first show that $\mathrm{Cov}(\Theta_i, \Theta_{i+k}) = \mathcal{O}(1/k)$. Let $g(x) = \min\{x^+, C_{\max}\}$ and $h(x) = \min\{x^-, D_{\max}\}$. Then, $\mathrm{Lip}(g) = \mathrm{Lip}(h) = 1$. Consider

$$\begin{aligned}&|\mathrm{Cov}(\Theta_i, \Theta_{i+k})| \\ &\quad= |\mathrm{E}[(\Theta_i - \mathrm{E}[\Theta_i])(\Theta_{i+k} - \mathrm{E}[\Theta_{i+k}])]| \\ &\quad\leq \left(\alpha_c + \frac{1}{\alpha_d}\right)^2 \kappa_k = \mathcal{O}\left(\frac{1}{k}\right).\end{aligned}$$

For unlimited $S_{\max}$, we have

$$C_i = \min\{\Delta_i^+, C_{\max}\}, \quad S_{i+1} = \max\{S_i + \Theta_i, 0\}.$$

Consider

$$\begin{aligned}\sum_{i=1}^n |\Delta_i - C_i + D_i| &= \sum_{i=1}^n ((-\Delta_i + C_i - D_i) + 2[\Delta_i - C_i + D_i]^+) \\ &= \sum_{i=1}^n (-\Delta_i + (1-\alpha)C_i + 2[\Delta_i - C_{\max}]^+) \\ &\quad + \alpha_d(S_{n+1} - S_1) \\ &= \sum_{i=1}^n (|\Delta_i| - (1+\alpha)\min\{\Delta_i^+, C_{\max}\}) \\ &\quad + \alpha_d(S_{n+1} - S_1)\end{aligned}$$

where the last equality follows by $\Delta_i^+ = [\Delta_i - C_{\max}]^+ + \min\{\Delta_i^+, C_{\max}\}$. By Lemma 2, we have

$$\begin{aligned}&\lim_{n\to\infty} \frac{\alpha_d \mathrm{E}[S_{n+1} - S_1]}{n} \\ &\quad= [\mathrm{E}[\alpha \min\{\Delta^+, C_{\max}\} - \min\{\Delta^-, D_{\max}\}]]^+\end{aligned}$$

and thus

$$\begin{aligned}&\mathcal{J}(\pi^*, S_1) \\ &\quad= \mathrm{E}[|\Delta|] - (1+\alpha)\mathrm{E}[\min\{\Delta^+, C_{\max}\}] \\ &\qquad + \left[\mathrm{E}\left[\alpha \min\{\Delta^+, C_{\max}\} - \min\{\Delta^-, D_{\max}\}\right]\right]^+ \\ &\quad= \mathrm{E}[|\Delta|] - \mathrm{E}[\min\{\Delta^+, C_{\max}\}] \\ &\qquad - \min\{\alpha \mathrm{E}[\min\{\Delta^+, C_{\max}\}],\ \mathrm{E}[\min\{\Delta^-, D_{\max}\}]\}.\end{aligned}$$

■

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